Logarithmic function

Function $y = \log_a x$ is logarithmic function and it is the inverse function to $y = a^x$ $(a \neq 1, a > 0, a \in R)$.

$y = \log_a x$ $\longrightarrow$ (logarithm of x based on a)

If $a = e \rightarrow y = \ln x$

If $a = 10 \rightarrow y = \log x$

For basic logarithmic function is:

1) The functions are defined for $x \in (0, \infty)$

2) Graph cutting x-line at point A (1,0) $\longrightarrow$ Zero of function

3) i) If $a > 1$, function is growing
   ii) If $0 < a < 1$, function decrease

4) i) If $a > 1$ then: $y > 0$ for $x \in (1, \infty)$
   $y < 0$ for $x \in (0,1)$

   ii) If $0 < a < 1$ then: $y > 0$ for $x \in (0,1)$
   $y < 0$ for $x \in (1, \infty)$

Here are a few examples of the basic graphics:

1) $y = \log_2 x$

Values for x choose wisely and make a table: x=1,2,4,8, $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$.(We'll see why!)

For x=1 $\Rightarrow y = \log_2 1 = 0$

For x=2 $\Rightarrow y = \log_2 2 = 1$

For x=4 $\Rightarrow y = \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2 \cdot 1 = 2$

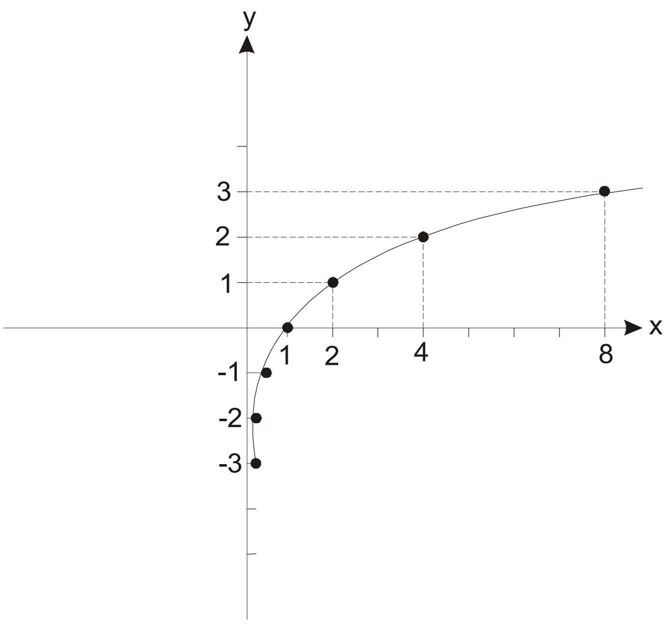
For x=8 $\Rightarrow y = \log_2 2^3 = 3\log_2 2 = 3 \cdot 1 = 3$

For x=$\frac{1}{2}$ $\Rightarrow y = \log_2 \frac{1}{2} = \log_2 2^{-1} = -1\log_2 2 = -1 \cdot 1 = -1$

For x= $\frac{1}{4}$ $\Rightarrow y=\log_2 \frac{1}{4}=\log_2 2^{-2}=-2$

For x= $\frac{1}{8}$ $\Rightarrow y=-3$

x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
y	-3	-2	-1	0	1	2	3



How is $a=2>0$ in $y=\log_2 x$ it is growing!

2) $y=\log_{\frac{1}{2}} x$

x=1 $\Rightarrow y=-\log_2 1=0$

x=2 $\Rightarrow y=-\log_2 2=-1$

x=4 $\Rightarrow y=-\log_2 4=-\log_2 2^2=-2\log_2 2=-2\cdot 1=-2$

x=8 $\Rightarrow y=-\log_2 2^3=-3\log_2 2=-3\cdot 1=-3$

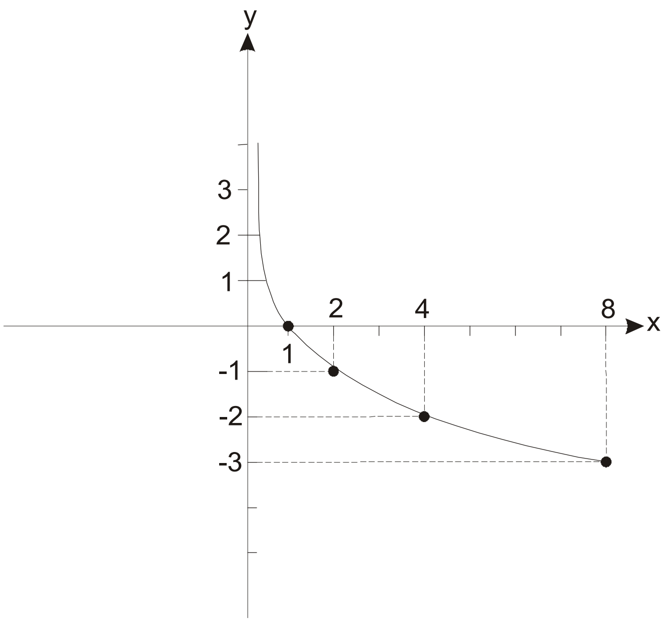
x= $\frac{1}{2}$ $\Rightarrow y=-\log_2 \frac{1}{2}=-\log_2 2^{-1}=-(-1)\cdot\log_2 2=+1\cdot 1=1$

x= $\frac{1}{4}$ $\Rightarrow y=-\log_2 \frac{1}{4}=-\log_2 2^{-2}=-(-2)=2$

x= $\frac{1}{8}$ $\Rightarrow y=3$

Similar , make a table, choose wisely!

x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
y	3	2	1	0	-1	-2	-3



When the basis is between 0 and 1, figure is decreasing!

3) We have function: $y = \log_a(3x^2 - 2x) \quad (a > 0, a \neq 1)$

i) Where the function is defined?
ii) Graph cutting x-line to point …. Where?
iii) Determine x ,so that for basis $a = \sqrt{5}$ value of function is 2.

Solution:

Take *heed:* All the *behind the log must be > 0*

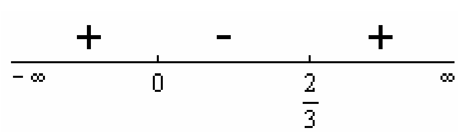
i) $3x^2 - 2x > 0$

$$3x^2 - 2x = 0$$

$$x_{1,2} = \frac{2 \pm 2}{6}$$

$$x_1 = 0$$

$$x_2 = \frac{2}{3}$$



$$x \in (-\infty, 0) \cup \left(\frac{2}{3}, \infty\right)$$

The area of definition is $x \in (-\infty, 0) \cup \left(\frac{2}{3}, \infty \right)$

ii) $\log_a (3x^2 - 2x) = 0$ How is $\log_a 1 = 0$ it must be:

$$3x^2 - 2x = 1$$
$$3x^2 - 2x - 1 = 0$$
$$x_{1,2} = \frac{2 \pm 4}{6}$$
$$x_1 = 1$$
$$x_2 = -\frac{1}{3}$$

$x_1 = 1$ and $x_2 = -\frac{1}{3}$

iii) $y = \log_a (3x^2 - 2x) = 0$ $\left. \begin{matrix} a = \sqrt{5} \\ y = 2 \end{matrix} \right\}$ change

$$\log_{\sqrt{5}} (3x^2 - 2x) = 2$$

Go by the definition : $\log_A B = \otimes \Leftrightarrow B = A^{\otimes}$

$$3x^2 - 2x = \sqrt{5}^2$$
$$3x^2 - 2x = 5$$
$$3x^2 - 2x - 5 = 0$$
$$x_{1,2} = \frac{2 \pm 8}{6}$$
$$x_1 = \frac{10}{6} = \frac{5}{3}$$
$$x_2 = \frac{-6}{6} = -1$$

As the area of definition is $x \in (-\infty, 0) \cup \left(\frac{2}{3}, \infty \right)$ only both solutions are “good”.

3) Find zero function $y = \log_3\left(\sqrt{x^2+21} - \sqrt{x^2+12}\right)$

Solution:

Condition is:

$$\sqrt{x^2+21} - \sqrt{x^2+12} > 0$$

This applies , because

$$\sqrt{x^2+12+7} - \sqrt{x^2+12} > 0 \Leftrightarrow \sqrt{x^2+12+7} > \sqrt{x^2+12}$$

$$\log_3\left(\sqrt{x^2+21} - \sqrt{x^2+12}\right) = 0$$
$$\sqrt{x^2+21} - \sqrt{x^2+12} = 1$$

$$\sqrt{x^2+21} = \sqrt{x^2+12} + 1 .... /()^2$$
$$x^2 + 21 = \left(\sqrt{x^2+12} + 1\right)^2$$
$$\cancel{x^2} + 21 = \cancel{x^2} + 12 + 2\sqrt{x^2+12} + 1$$
$$2\sqrt{x^2+12} = 21 - 12 - 1$$
$$2\sqrt{x^2+12} = 8$$
$$\sqrt{x^2+12} = 4 .... /()^2$$
$$x^2 + 12 = 16$$
$$x^2 = 4 \rightarrow x = \pm\sqrt{4} \rightarrow \boxed{x_1 = -2} \wedge \boxed{x_2 = 2}$$